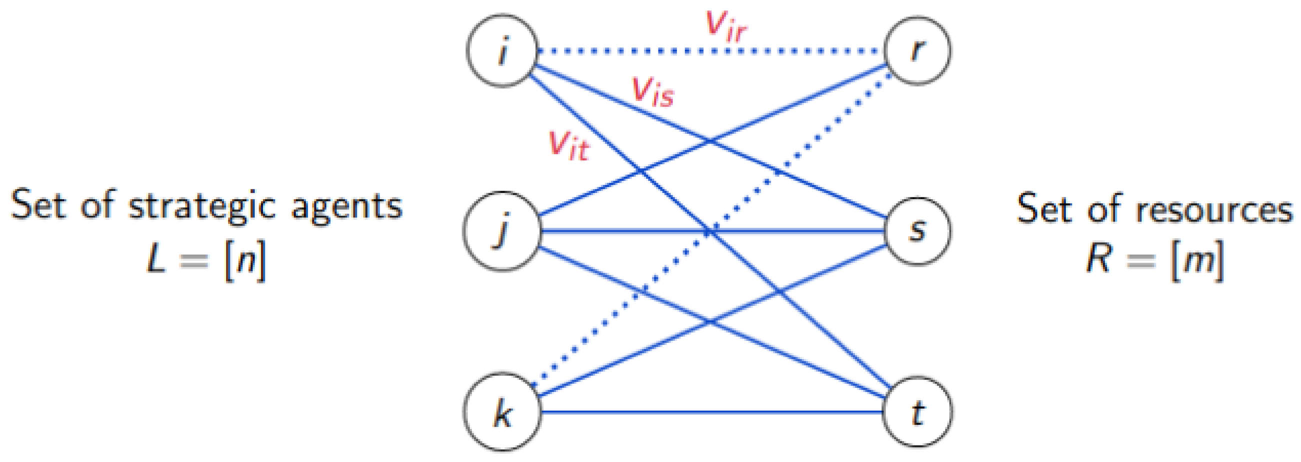
Assignment Mechanisms with Predictions in the Private Graph Model

Riccardo Colini-Baldeschi[1], Sophie Klumper[2,3], Guido Schäfer[2,3], Artem Tsikiridis[2]

rickuz@meta.com, {sophie.klumper, guido.schaefer, artem.tsikiridis}@cwi.nl

[1]Central Applied Science, Meta, UK
[2]Networks and Optimization, Centrum Wiskunde & Informatica (CWI), NL
[3]Institute for Logic, Language and Computation, University of Amsterdam, NL

Bipartite Matching Problem (BMP) in Private Graphs



- The value $v_{ij} \in \mathbb{R}^+$ is **public** information $\forall i, \forall j$
- Each agent i has a **private** compatibility set $E_i \subseteq \{i\} \times R$
- Each agent i declares a compatibility set $D_i \subseteq \{i\} \times R$
- Given D, we want a strategyproof mechanism $\mathcal{M}(D) = M$ to compute a feasible assignment $M \subseteq D$ of maximum value
- Each agent i wants to maximize their utility:

$$u_i(D) = \begin{cases} v_{ij} & \text{if } \exists (i,j) \in \mathcal{M}(D) \cap E_i \\ 0 & \text{otherwise} \end{cases}$$

- A mechanism $\mathcal{M}$ is *strategyproof* if $\forall i \in L$, for any instance, any D_{-i} and any D'_i:

$$u_i(E_i, D_{-i}) \geq u_i(D'_i, D_{-i}).$$

- Dughmi and Ghosh [**DG10**] give a greedy deterministic mechanism that is strategyproof and 2-approximate, and prove it is best possible

BMP in Private Graphs with Predictions

- Additionally, we are given a *predicted feasible assignment* $\hat{M} \subseteq L \times R$
- Given D, $\hat{M}$ is **perfect** if $v(\hat{M} \cap D) = v(M^*_D)$, with M^*_D the optimal assignment in $G[D]$
- $\mathcal{M}$ is α**-consistent [LV21]**, $\alpha \geq 1$, if for every instance with a perfect prediction the assignment $M = \mathcal{M}(D)$ satisfies:

$$\alpha \cdot v(M) \geq v(M^*_D)$$

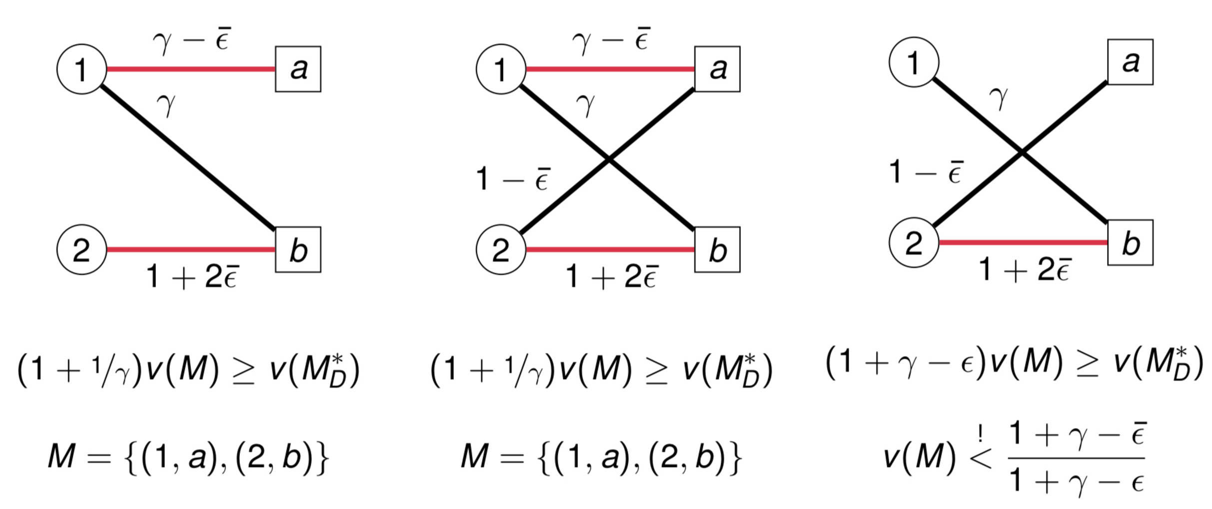
- $\mathcal{M}$ is β**-robust [LV21]**, $\beta \geq 1$, if for every instance with an arbitrary prediction, the assignment $M = \mathcal{M}(D)$ satisfies:

$$\beta \cdot v(M) \geq v(M^*_D)$$

Theorem

Let $\gamma > 1$ be fixed arbitrarily. Then no deterministic strategyproof mechanism can achieve $(1 + \frac{1}{\gamma})$-consistency and $(1 + \gamma - \epsilon)$-robustness for any $\epsilon > 0$.

Proof by contradiction: assume there exists such $\mathcal{M}$. Consider $\gamma > 1$, $\bar{\epsilon} > 0$ and $\hat{M} \cap D$:



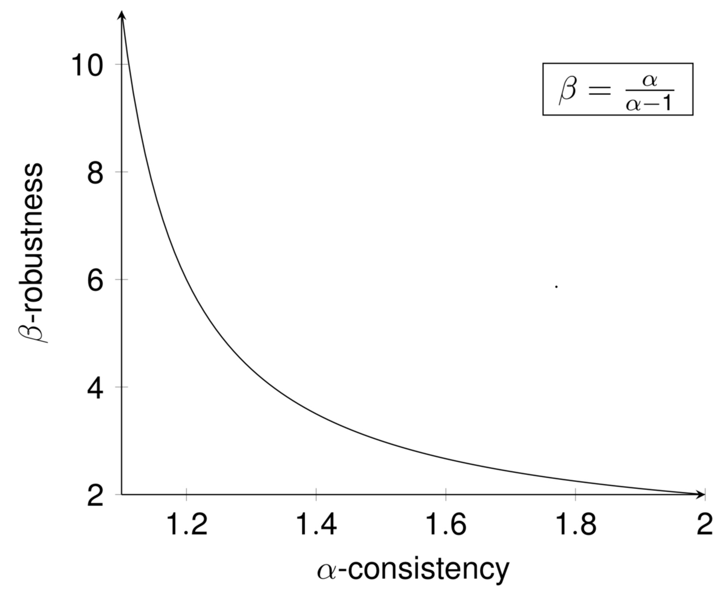


Figure 1: No deterministic strategyproof mechanism can achieve the consistency-robustness combinations below the curve.

Optimal Deterministic Mechanism for BMP

Our mechanism Boost is inspired by the deferred acceptance alg. by Gale & Shapley (1962):

- *Agent proposal order:* Each agent i maintains an order on their set of incident edges D_i by sorting them according to non-increasing values v_{ij}
- *Resource preference order:* Each resource j maintains an order on their set of incident edges $D_j = \{(i,j) \in D\}$ by sorting them according to non-increasing offer values θ_{ij}
- Key idea: the offer of agent i when proposing to j is *boosted* if the edge $(i,j) \in \hat{M}$, i.e.,

$$\theta_{ij}(\gamma, \hat{M}) = \begin{cases} v_{ij} & \text{if } (i,j) \notin \hat{M}, \\ \gamma \cdot v_{ij} & \text{if } (i,j) \in \hat{M}. \end{cases}$$

Optimal Deterministic Mechanism for BMP (cont.)

Mechanism 0: Boost

```
M = ∅
P_i = list of incident edges D_i ordered by decreasing v_ij for each i ∈ L
while there exists an active agent i do
    i proposes θ_ij = θ_ij(γ, M̂) to next resource j on their list P_i
    if offer θ_ij is larger than j's current best offer then
        j rejects their current tentative mate k (if any): M = M \ {(k, j)}
        j tentatively accepts i as their new mate: M = M ∪ {(i, j)}
    remove (i, j) from proposal list P_i
return M
```

We define the **prediction error** $\eta(\mathcal{I}) \in [0,1]$ of an instance $\mathcal{I}$ as

$$\eta(\mathcal{I}) = 1 - \frac{v(\hat{M} \cap D)}{v(M^*_D)}$$

Theorem

Fix some error parameter $\hat{\eta} \in [0,1]$. Consider the class of instances of BMP in the private graph model with prediction error at most $\hat{\eta}$. Then, for every confidence parameter $\gamma \geq 1$, Boost is group-strategyproof and has an approximation guarantee of

$$g(\hat{\eta}, \gamma) = \begin{cases} \frac{1+\gamma}{\gamma(1-\hat{\eta})} & \text{if } \hat{\eta} \leq 1 - \frac{1}{\gamma}, \\ 1 + \gamma & \text{otherwise.} \end{cases}$$

In particular, Boost is $(1 + \frac{1}{\gamma})$-consistent and $(1 + \gamma)$-robust, which is best possible.

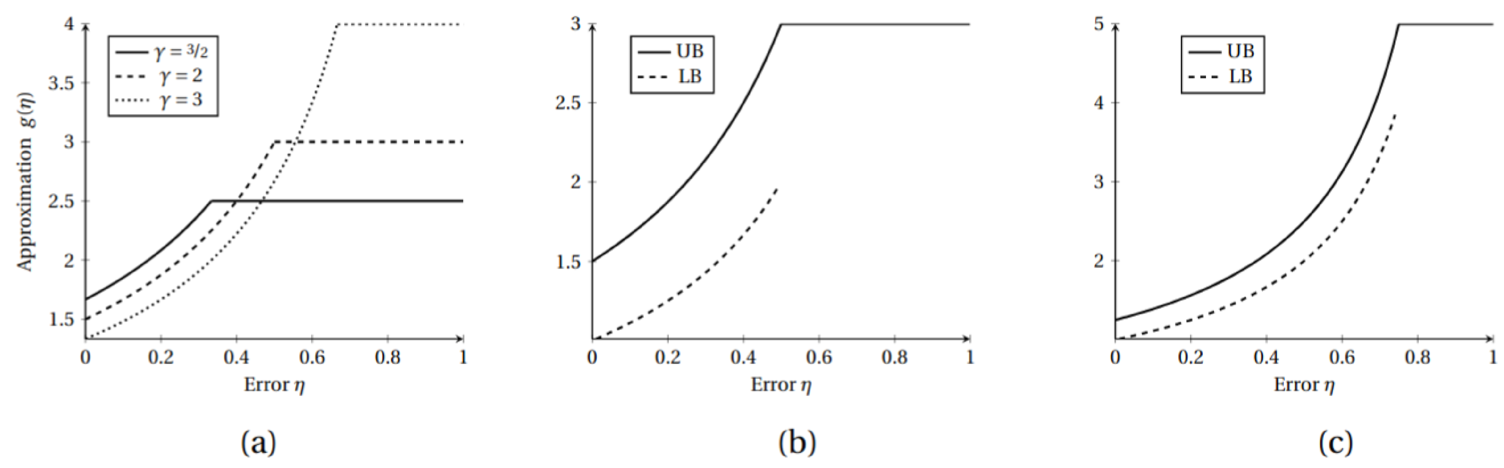


Figure 2: Approximation guarantee $g(\hat{\eta})$ as a function of η. (a) For $\gamma \in \{\frac{3}{2}, 2, 3\}$, (b) upper vs. lower bound for $\gamma = 2$ and (c) upper vs. lower bound for $\gamma = 4$.

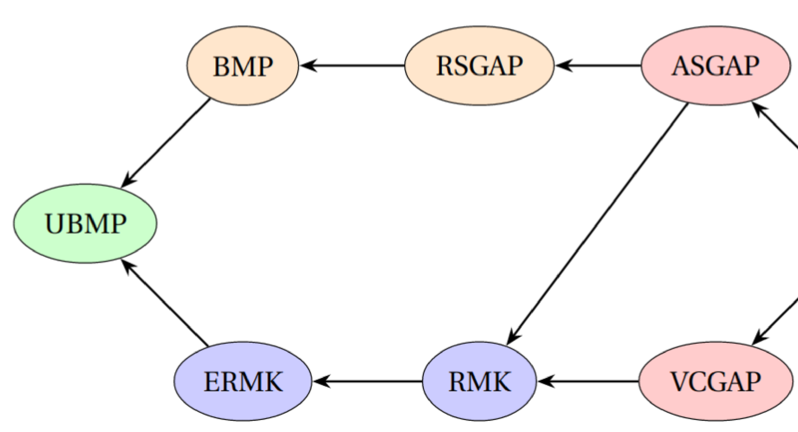
Randomized Mechanisms for GAP variants

Bipartite Matching Problem:

- Our mechanism Boost-or-Trust runs Boost with parameter $\delta(\gamma) = \sqrt{2(\gamma + 1)} - 1$ with probability p and returns $\hat{M} \cap D$ with probability $1 - p$, with $p = 2/(\delta(\gamma) + 1)$
- Boost-or-Trust is universally GSP, $(1 + \frac{1}{\gamma})$-consistent and $\sqrt{2(\gamma + 1)}$-robust.

Table 1: Overview of GAP variants considered in our paper.

GAP Variant	**Restrictions** ($\forall i \in L, \forall j \in R$)
Unweighted Bipartite Matching (UBMP)	$v_{ij} = 1, s_{ij} = 1, C_j = 1$
Bipartite Matching Problem (BMP)	$s_{ij} = 1, C_j = 1$
Restricted Multiple Knapsack (RMK)	$v_{ij} = v_i, s_{ij} = s_i$
Equal RMK (ERMK)	$v_{ij} = s_{ij} = v_i$
Value Consensus GAP (VCGAP)	$\exists \sigma : v_{i\sigma(1)} \geq \ldots, \geq v_{i\sigma(m)}$
Agent Value GAP (AVGAP)	$v_{ij} = v_i$
Resource Value GAP (RVGAP)	$v_{ij} = v_j$
Agent Size GAP (ASGAP)	$s_{ij} = s_i$
Resource Size GAP (RSGAP)	$s_{ij} = s_j$



- Our mechanism Greedy orders all declared edges according to a specific ranking (part of the input) and greedily adds edges in this order to an initially empty assignment (maintaining feasibility)
- Greedy coupled with an arbitrary ranking may not result in a strategyproof mechanism for the GAP in general!
- Our mechanism for ERMK randomizes over Greedy and $\hat{M} \cap D$. This leads to universal GSP, $(1 + \frac{1}{\gamma})$-consistency and $\frac{1}{2}(\sqrt{12\gamma + 13} + 1)$-robustness.
- Our mechanisms for ASGAP and VCGAP randomize over Greedy, Boost and $\hat{M} \cap D$. This leads to universal GSP, $(1 + \frac{3}{\gamma})$-consistency and $(3 + \gamma)$-robustness.

References

[**DG10**] Shaddin Dughmi and Arpita Ghosh. Truthful assignment without money. EC 2010
[**LV21**] Thodoris Lykouris and Sergei Vassilvitskii. Competitive caching with machine learned advice. JACM 2021